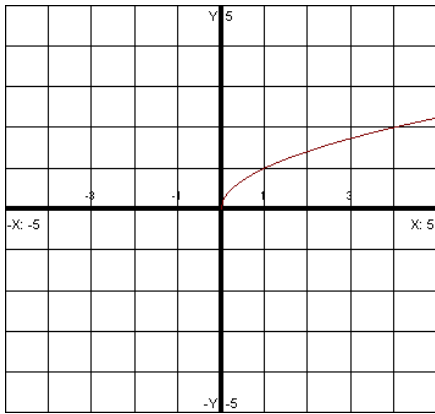


2.5 Transformations of Functions

We will first look at the major graphs you should know how to sketch:



$$y = \sqrt{x}$$

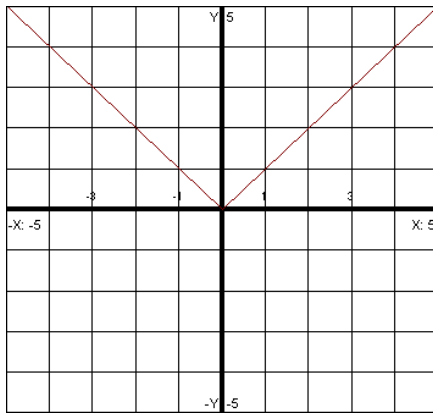
Square Root Function

Domain: $[0, \infty)$

Range: $[0, \infty)$

Increasing: $(0, \infty)$

Decreasing: None



$$y = |x|$$

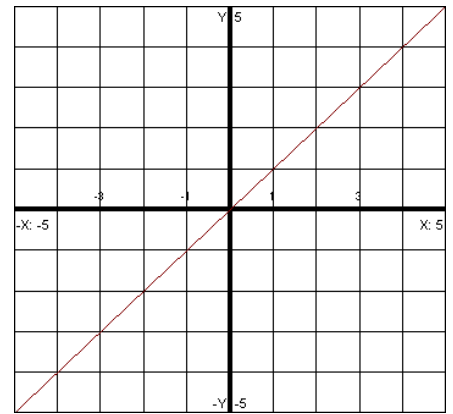
Absolute Value Function

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$



$$y = x$$

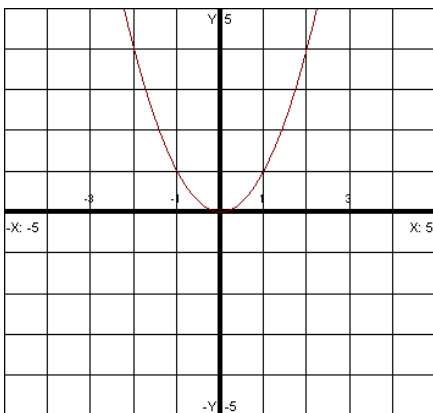
Identity Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, \infty)$

Decreasing: None



$$y = x^2$$

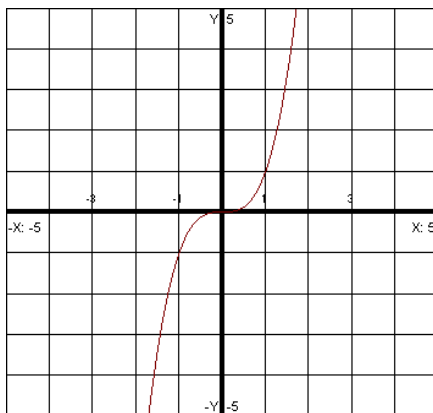
Standard Quadratic Function

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$



$$y = x^3$$

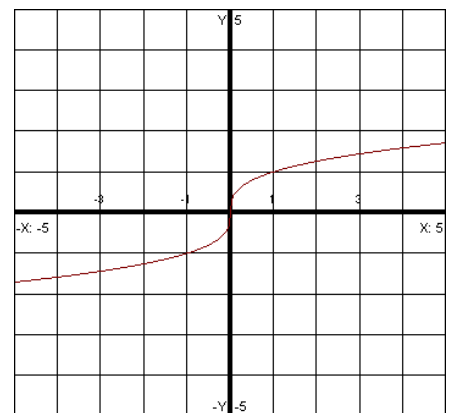
Standard Cube Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, \infty)$

Decreasing: None



$$y = \sqrt[3]{x}$$

Cube Root Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, \infty)$

Decreasing: None

Transformations and Graph Sketches

When we used to graph a line the usual thing to do was make a table of values and plot the points. This method works but takes a long time. Transformations allows you move a graph up or down, left or right into a new position. We start with the basic graphs we learned in the last section and will move it based on the following criteria.

Suppose $y = f(x)$ is the original function (one we looked at in the previous section)

$Y = f(x) + k$ moves $f(x)$ k units up

$Y = f(x) - k$ moves $f(x)$ k units down

$Y = f(x - h)$ moves $f(x)$ h units to the right

$Y = f(x + h)$ moves $f(x)$ h units to the left

$Y = -f(x)$ flips the graph over a horizontal axis

$Y = f(-x)$ flips the graph over a vertical axis

$Y = a(f(x))$ If $|a| > 1$ then there is a vertical stretch. If $0 < |a| < 1$, then there is a vertical compression.

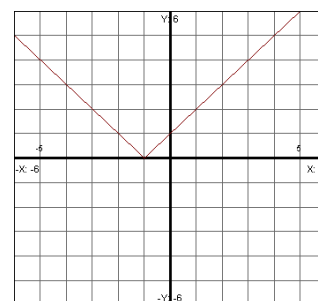
Let's look at some examples. For all of these we are just making a sketch of the function.

EXAMPLE: Sketch $y = |x + 1|$ by using transformations.

First we need to recognize what kind of graph we are using. This is the absolute value graph which is a V shaped graph centered at the origin. Since there is a $+1$ inside the absolute value we are looking at rule 4 which says we will move the graph 1 unit to the left. So the graph will look like:

What is the x-intercept? It crosses the x-axis at $(-1, 0)$.

What is the y-intercept? It crosses the y-axis at $(0, 1)$.

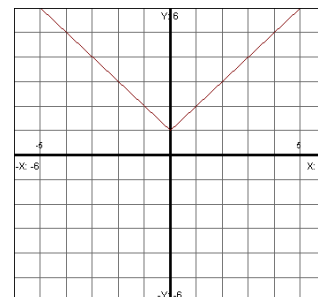


EXAMPLE: Sketch $y = |x| + 1$ by using transformations.

This may look similar to the graph above, but it is different because the 1 is on the outside of the absolute values. This is rule 1 which says the graph will move 1 unit up:

There are no x-intercepts on this one.

The y-intercept is $(0, 1)$.

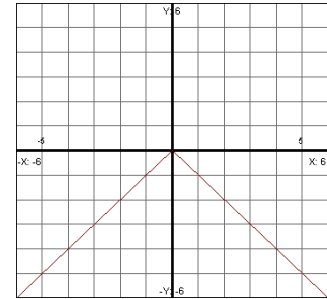


EXAMPLE: Sketch $y = -|x|$ by using transformations.

This one has a negative on the outside of the absolute value. Since the negative is outside we will use rule 5 which says the graph will flip over the horizontal axis. This will flip the graph upside down and the pivot point is the origin $(0, 0)$:

x-int: $(0, 0)$

y-int: $(0, 0)$

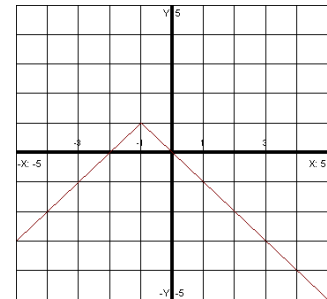


EXAMPLE: Sketch $y = -|x + 1| + 1$ by using transformations.

Now let's put it all together. Usually you will be using more than one transformation. This problem does puts the last three examples together. We start with our absolute value graph at the origin. We will move it one place to the left, then up one unit and then flip it upside down:

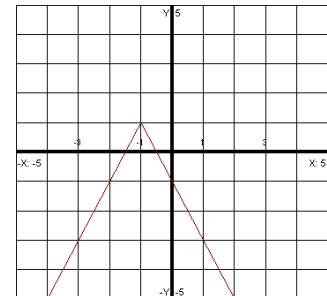
x-intercept: $(-2, 0)$ and $(0, 0)$

y-intercept: $(0, 0)$



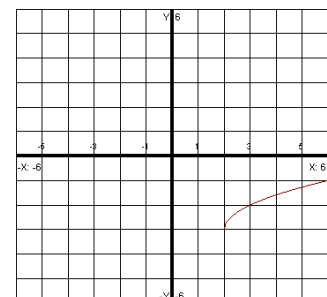
EXAMPLE: Sketch $y = -2|x + 1| + 1$ by using transformations

You will move the graph in the same directions as the last example, but now there is a number in front of the absolute value. This doubles all of the regular y-values, causing the graph to be vertically stretched.



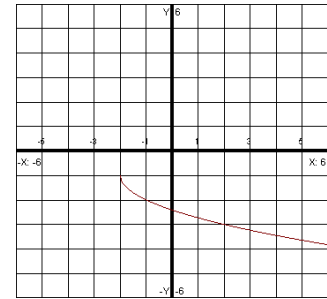
EXAMPLE: Sketch $y = \sqrt{x - 2} - 3$ by using transformations.

Now we will look at a different type of graph. This one is the square root. Our transformations now tell us we will move the square root graph 2 places to the right and 3 units down. We don't need to flip the graph because there is no negative outside of the function, or inside next to an x.



EXAMPLE: Sketch $y = -\sqrt{x+2} - 1$ by using transformations.

This one requires us to move the square root graph 2 places to the left and down one unit. Once this is done we need to flip the graph over the horizontal axis because the negative is on the outside of the equation.



EXAMPLE: Sketch $y = \sqrt{3-x} + 2$ by using transformations.

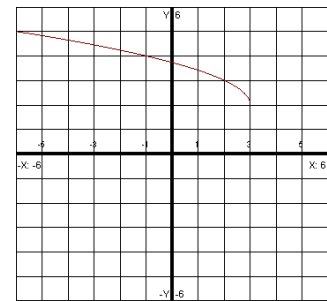
In order to use the transformation rules the x must come first and there must be a one in front of x . In our problem above we need to first put the x first and then we will factor out a negative:

$$y = \sqrt{3-x} + 2$$

$$y = \sqrt{-x+3} + 2 \quad \text{Here we put the } x \text{ term first}$$

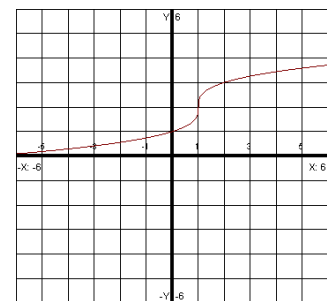
$$y = \sqrt{-(x-3)} + 2 \quad \text{Here we factored out a } -1.$$

Now we are ready to graph. Since we already factored out the negative we need to move the graph 3 places to the right and then up 2 units. There is a negative inside the function, so we need to use rule 6 of the transformations which says we will flip the graph over the vertical axis.



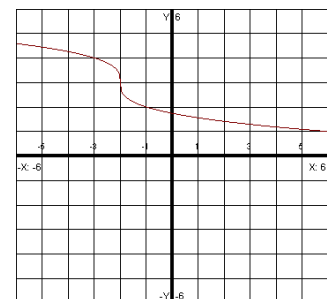
EXAMPLE: Sketch $y = \sqrt[3]{x-1} + 2$ by using transformations.

Be careful you don't confuse this with the square root graphs we just did. This one looks different. We will move the regular cube root graph to the right 1 unit and up 2 units.



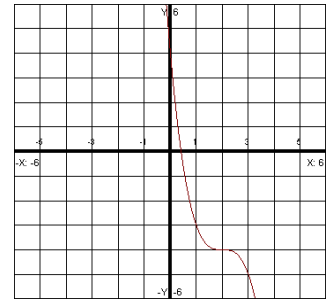
EXAMPLE: Sketch $y = \sqrt[3]{-(x+2)} + 3$ by using transformations.

For this one since the negative is already factored out we will move the graph 2 units to the left and up 3 units. We will also flip the graph over the vertical axis since the negative is inside the radical.



EXAMPLE: Sketch $y = -(x - 2)^3 - 4$ by using transformations.

Here we have the cube graph. Don't confuse it with the cube root graph. We need to move the graph 2 units to the right and down four units. There is a negative outside so we will flip the graph over the horizontal axis.



EXAMPLE: Sketch $y = (2 - x)^3 - 4$ by using transformations.

For this one we need to once again put the x term first and then factor out the negative sign.

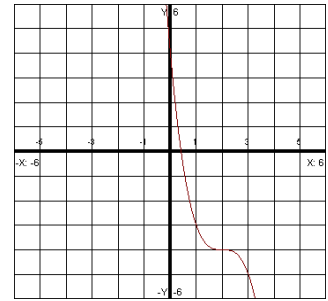
$$y = (2 - x)^3 - 4$$

$$y = (-x + 2)^3 - 4$$

$$y = -(x - 2)^3 - 4$$

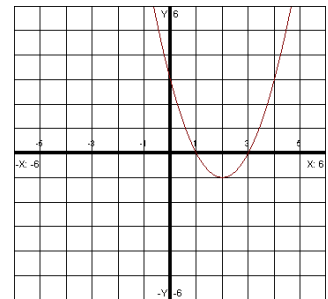
So we move the graph 2 places to the right and down 4 units. The negative is inside the function, so we need to flip it over the vertical direction.

Does this graph look familiar? It is the same graph as the previous problem. Why? Notice the negative inside the function above. If I raise a negative to an odd power then the negative can come outside the parenthesis. If this happens you will get the same equation as the last example.



EXAMPLE: Sketch $y = (x - 2)^2 - 1$ by using transformations.

Now we have the squaring function. This will be a parabola. We will move the parabola 2 places to the right and down 1 unit.



EXAMPLE: Sketch $y = -(x - 2)^2 - 1$ by using transformations.

This one is shifted in the same directions as in the last example. We have a negative inside the function which means we will shift it over the vertical axis. If you flip the graph it will look the same as the original, so the graph is the same as the previous example. Also if you raise a negative to an even power the negative goes away and you will get the same equation as in the previous example so that is why the graph looks the same.

